THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 59, No. 7, September 1980
Printed in U.S.A.

Dimensioning Trunk Groups for Digital Networks

By W. B. ELSNER

(Manuscript received January 22, 1980)

The use of digital switching and transmission facilities in the metropolitan environment is expanding rapidly. Since these facilities must be installed in multiples of 24 circuits, their per-trunk cost is nonlinear. Traditional methods for dimensioning high-usage trunk groups, however, assume linear facility costs and thus lead to non-optimal network designs. An approximate method for dealing with nonlinear trunk costs is modular engineering, i.e., the sizing of trunk groups in multiples of a fixed module size. We examine the cost benefit of modular engineering for digital networks by simulating the traffic and facility provisioning processes for a representative city. Results show that sizing one-way high-usage groups in modules of 12 trunks and two-way groups in modules of 24 saves 4 to 9 percent of total network first cost relative to the nonmodular alternative. Administrative costs are also reduced through a decrease in the number of high-usage groups. The results are robust to network structure and growth, facility costs, forecast error, and accelerated switch exhausts.

I. INTRODUCTION AND SUMMARY

1.1 Background

The predominant transmission facility in metropolitan networks today is T-carrier, a digital, time-division multiplexed system that carries 24 voice-frequency channels on two pairs of wires. Its use is growing rapidly as its cost relative to other transmission systems continues to decrease and as digital switches, such as the No. 4 ESS, are deployed.

New digital terminals for the No. 1/1A ESS (the Digital Carrier Trunk, or DCT) and No. 4 ESS (the Digital Interface Frame, or DIF) require that trunk groups that terminate on either of these switches be provided on separate T-carrier systems (or "digroups"). This facility segregation implies that the fixed, per-system costs of the digroups and

terminals must be borne by the specific trunk groups which use them. The per-trunk costs of these groups are therefore nonlinear. Traditional methods for the economical sizing of high-usage groups in alternate-route trunk networks, however, assume that the costs of facilities are linear functions of the numbers of trunks. The use of these classical techniques may thus lead to nonoptimal network designs.

A heuristic approach for accounting for the nonlinear trunk costs is to constrain the sizes of high-usage groups to be multiples of a fixed size (module); we call such an approach "modular engineering." This method is already in use in the Bell System long-distance network, for which all groups are sized in multiples of 12 trunks.[1,2] In this paper, we examine whether modular engineering reduces the total cost for trunk networks in a T-carrier environment.

1.2 Summary of results

A computer simulation of the traffic and facility engineering processes for the Minneapolis/St. Paul network shows that the modular engineering of high-usage trunk groups carried on segregated digital facilities saves 4 to 9 percent of total network first cost relative to the nonmodular alternative. The optimal module sizes are 12 trunks for one-way groups and 24 trunks for two-way groups. The savings are the result of reductions on the order of 20 percent in the requirements for dedicated digroups which more than offset a 9-percent increase in the cost of switching.

Administrative costs are also reduced, through a 26-percent decrease in the total number of trunk groups. However, these savings are too small to influence the choice of module sizes. Similarly, the impact of possible switch exhausts due to the increase in tandem switching is shown to have no influence on the modular engineering decision.

The results are shown to be robust to changes in network structure (number and type of tandem switches, homing strategies, etc.), to network growth and evolution over a period of 13 years, to variations up to ±50 percent in facility costs, and to forecast error with coefficients of variation up to 30 percent.

II. PROBLEM STATEMENT

2.1 Cost-ratio engineering and multiplexed transmission facilities

The ultimate cost of a telephone network depends in part on the cost of the actual transmission and switching facilities that must be installed to realize the traffic network. The design of a minimum-cost network begins with the design of a trunk network that seeks to minimize an *assumed* facility cost. The success of this approach clearly depends on the accuracy of the cost assumptions.

The method currently in use in the Bell System for computing the optimal size of high-usage groups is ECCS Engineering.[3] In the early 1980s, this method will be replaced by Multihour Engineering,[4,5] a refinement of the ECCS method that recognizes that the maximum loads on the direct and alternate routes may occur at different times. Both methods rely on the crucial assumption that the cost of a trunk group is proportional to the number of trunks. The resulting high-usage group sizes depend on, among other things, the ratios of the per-trunk costs of the alternate and direct routes. Hence we refer to these techniques as "cost-ratio engineering" methods.

The cost of multiplexed transmission facilities is generally not linear. Such facilities are provided on a modular basis, i.e., some equipment is common to a fixed number of circuits (a module). In addition, there may or may not be equipment provided individually for each active circuit.

The cost of providing facilities for n circuits can thus be expressed by

$$\begin{aligned} c(n) &= P([(n-1)/N] + 1) + qn \\ c(0) &= 0 \end{aligned} \tag{1}$$

where

P = fixed cost (per module)
q = incremental cost (per circuit)
$[\cdot]$ = integer part of $\cdot$
N = module size.

This function is shown in Fig. 1. Actually, eq. (1) is only an approximation to the actual cost. There are usually many levels of modularity—a number of multiplexed transmission systems may share the same cable; a number of cables may share the same conduit, etc. For the purpose of this study, however, we restrict our attention to the nonlinear effects as expressed by eq. (1).

To satisfy the linearity requirement of cost-ratio engineering, some straight-line approximation to eq. (1) is required. One possibility is an "average, fully allocated" cost, in which each circuit is assigned an equal fraction of the total cost of a fully equipped (100-percent filled) module:

$$c'(n) = P(n/N) + qn. \tag{2}$$

This function is shown as a dashed line in Fig. 1. Note that, for any particular value of n, the difference between the solid curve and the dashed curve is the cost of the unused capacity of the common equipment. The validity of eq. (2) depends on the manner in which trunks are distributed in a module. In the next two sections, we examine this validity for metropolitan, digital transmission systems.

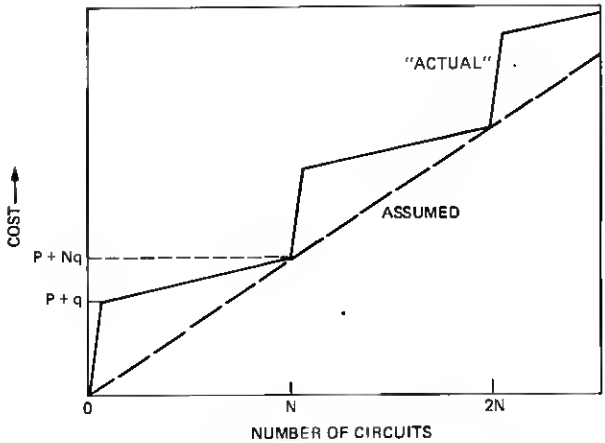


Fig. 1—Cost of multiplexed facilities.

2.2 The T-carrier environment—today

A typical *T*-carrier configuration prior to the introduction of digital switches and the new digital terminals is shown in Fig. 2. The digital-to-analog conversion and multiplexing functions are performed by *D*-type channel banks and channel units (CU). One *D*-bank is required at each end of a digroup and one CU is needed at each end for each active circuit. The 24 channels may have different origins or destinations in the building. Any 24 voice-frequency signals destined for a common distant building can share a digroup through appropriate interconnection at an Intermediate Distributing Frame (IDF). They could include trunks from different groups, special-service circuits, and even circuits that do not terminate in tbe building at all but are simply cross-connected for distribution to other destinations.

The ability of a carrier system to be shared by many different circuits has two important effects:

(*i*) The pool of circuits which can fill the system is large, and hence the proportion of unused capacity attributable to "breakage" is small.

(*ii*) As long as the circuits have access to all channels, the cost of any spare capacity is uniformly spread among all circuits, and the use of a linear approximation to the cost is reasonable.

2.3 The T-carrier environment—future

The two new digital terminals are illustrated in Fig. 3. One is the Digital Interface Frame (DIF), which directly interfaces a digroup with the No. 4 ESS digital switch. The other is the Digital Carrier Trunk

(DCT), which provides a direct interface between a digroup and the No. 1 or No. 1A ESS, by combining tbe functions of the channel unit and the 1 ESS trunk circuit into a single "combined" channel unit (CCU).

The salient feature of these new terminals, aside from their cost advantage over alternative *T*-carrier/ESS interfaces, is that they require that all 24 channels of a digroup be dedicated to the associated switch. (Note the absence of a distribution frame in Fig. 3.) Consequently, the pool of circuits that can share a common DIF- or DCT-terminated digroup is limited to one of the following:

(*i*) Trunks in groups to or from a single distant switch.
(*ii*) Trunks in groups to or from collocated distant switches that do not use DIF or DCT (this may include trunks to or from a switch in a third building, which are provided through cross-connection at an IDF).

Since the circuits that can use a common digroup are limited, the amount of spare capacity due to breakage is increased over today's environment. Moreover, the cost of this spare capacity is directly attributable to specific trunk groups. In such cases, the linear cost model is not valid.

2.4 The modular engineering approach

To illustrate the principle of modular engineering, consider a high-usage group which is provided on a dedicated multiplexed facility with negligible per-circuit cost [$q \ll P$ in eq. (1)]. Clearly, it is always advantageous to connect all the trunks in a module; they are all paid for, anyhow, and they carry traffic that would otherwise overflow to

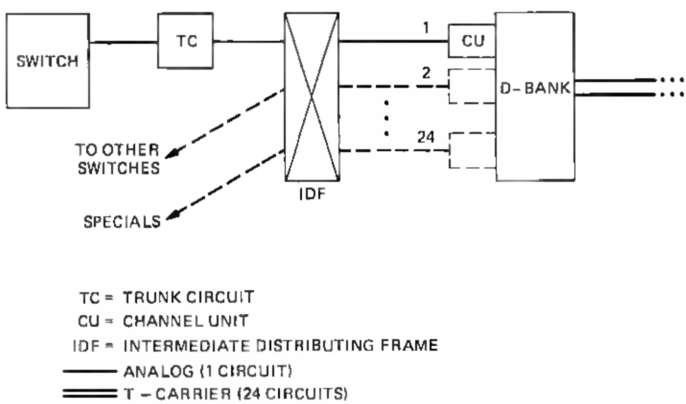


Fig. 2—Typical *T*-carrier interface—today.

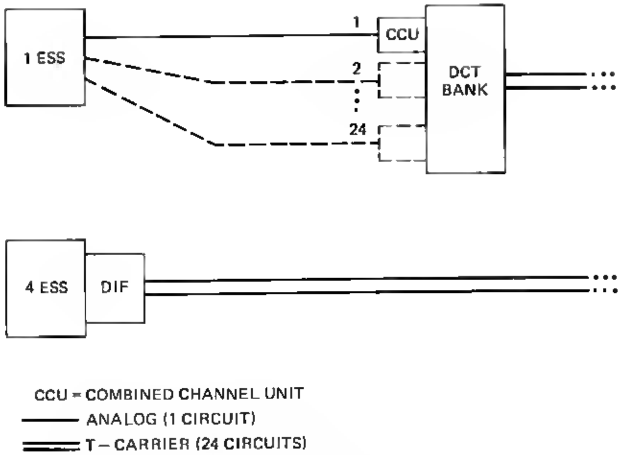


Fig. 3—T-carrier interfaces through DCT and DIF.

the alternate route. The objective of trunk engineering, in this case, is to determine the most economical number of modules of trunks in the high-usage group. The simplest way to accomplish this is first to obtain the cost-ratio solution and then to round this solution to a neighboring multiple of the module size, using an appropriate rounding threshold.

If the per-circuit cost is not negligible, modular engineering will not, in general, yield the optimal solution. For a sufficiently small ratio q/P, however, we expect modular engineering to yield more economical high-usage group sizes than cost-ratio engineering. One objective of the present study, then, is to determine the applicability of modular engineering for values of q and P typical of DCT- and DIF-terminated T-carrier.

III. THE STUDY MODEL

We examined the effects of various module sizes and rounding thresholds on the cost of a network by simulating the network engineering process for a particular metropolitan area. The basic flow of the simulation is shown in Fig. 4.

First, a trunk network is designed from a knowledge of the network geometry (physical location of end offices and tandems, homing configuration, etc.), the approximate, linearized cost of facilities, and the traffic demands. Standard cost-ratio engineering is used to determine preliminary trunk requirements; high-usage groups which are to be provided on DIF- or DCT-terminated T-carrier are then rounded up or down to modular sizes, according to the rules being tested. The actual cost of the facilities required is estimated by a separate program which specifically accounts for the nonlinearity of the cost of T-carrier

facilities that terminate on DIF or DCT. Thus, we can determine the modular engineering rules that result in the least-cost network.

The metropolitan area chosen for study was Minneapolis/St. Paul. The network consists of 74 end offices, 43 of which are No. 1 ESS, and a single tandem (No. 4 ESS) that switches both the intracity (local) traffic and the intercity (toll) traffic. End offices within the network are directly interconnected by high-usage groups, provided the economical sizes of these groups meet a prescribed minimum threshold (three trunks for nonmodular groups; one module, after rounding, for modular groups). The overflow from these high-usage groups is offered to final groups to the tandem. All final groups are engineered to 1-percent blocking.

Transmission facilities are assumed to be of two types: *T*-carrier, terminated as shown in Fig. 2 (for non-ESS offices) and Fig. 3 (for ESS offices and the tandem), and voice frequency (VF) cable pairs terminating directly on trunk circuits. The prove-in distance of *T*-carrier over VF ranges from 0 to 3 miles, depending on the types of terminations on either end. Trunk groups on non-DIF/DCT *T*-carrier are assumed to share the digroups with other trunk groups and/or special-service circuits, and are charged with average allocated costs.

IV. RESULTS

4.1 One-way groups

We consider first the network with all groups one-way (i.e., each trunk can be seized from one end only, and hence separate groups must be provided for each direction of traffic between two nodes). Figure 5 shows the total first cost for the network as a function of

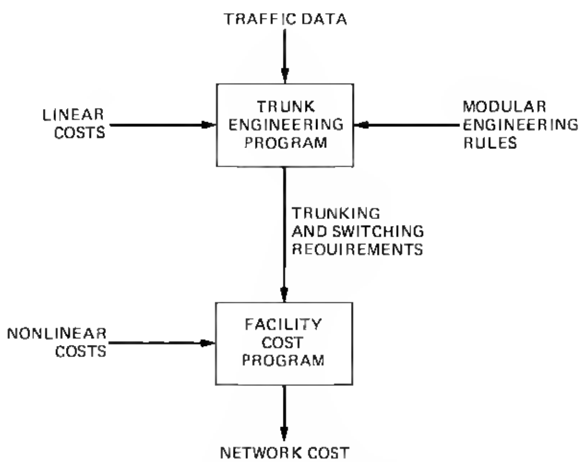


Fig. 4—Network engineering simulation.

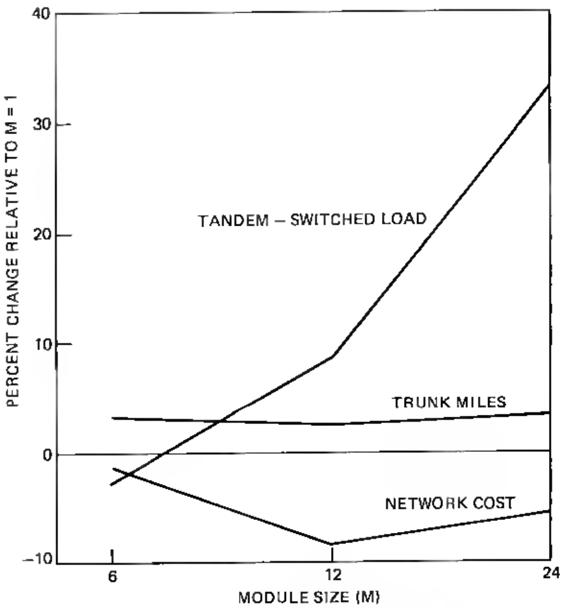


Fig. 5—Network cost, trunk miles, and tandem-switched load (one-way groups only).

three module sizes: 6, 12, and 24. In all cases, the round-up threshold is one-half module (i.e., 3, 6, and 12 trunks, respectively). The costs are shown relative to the nonmodular alternative and include all transmission and switching requirements for serving both the local and the toll traffic.

We observe first that modules of 24 trunks (the fundamental multiplex module for T-carrier) reduce the network cost by about 5 percent. This result indicates that the heuristic approach of sizing high-usage groups to fill the DIF- and DCT-terminated digroups indeed yields a net savings relative to cost-ratio engineering.

The more interesting result, however, is that a greater saving is obtained with modules of 12 trunks. This result is a consequence of the fact that pairs of one-way groups between common nodes share common DIFs and DCTs along with the associated digroups. A large portion (about 75 percent) of such pairs of one-way groups have aggregate modular sizes that are multiples of 24 trunks, and thus occupy fully utilized digroups. A further reduction of the module size to six trunks, however, proved to be less economical than modules of 12 or 24.

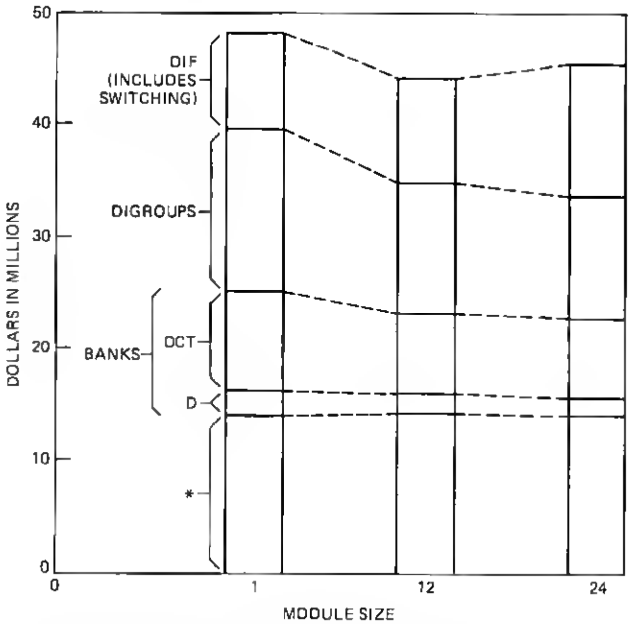
The reduction of network cost attributable to modular engineering occurs in spite of the increases in both circuit-miles and tandem-switched CCS, as indicated in Fig. 5. Clearly, the savings must be the

result of a more efficient utilization of the dedicated carrier systems and their terminals. Figure 6 shows the actual cost breakdown by major categories of facilities for no modular engineering, and for modules of 12 and 24 trunks, respectively. This figure shows that while the requirements for equipment that is provided on a per-circuit basis (trunk circuits, channel units, and VF cable pairs) remain relatively constant, the requirements for channel banks and digroups are reduced substantially—by about 20 percent with modules of 12 trunks, and by an additional 5 percent with modules of 24.

The savings in carrier systems and terminals are partially offset by the increase in the cost for tandem switch terminations, reflecting the increased switched loads shown in Fig. 5. (The cost of the No. 4 ESS is termination-dependent and is included in the cost of the DIFs.) Indeed, it is the large increase in tandem switch terminations associated with the increase in module size from 12 to 24 trunks that more than offsets the corresponding decrease in carrier costs, making modules of 12 the more economical choice.

4.2 Two-way groups

An important advantage of DIF and DCT is that they allow two-way trunking at no additional cost over one-way trunking. The opportunity



*VF CABLE, TRUNK CIRCUITS, AND CHANNEL UNITS

Fig. 6—Facility cost breakdown (one-way groups only).

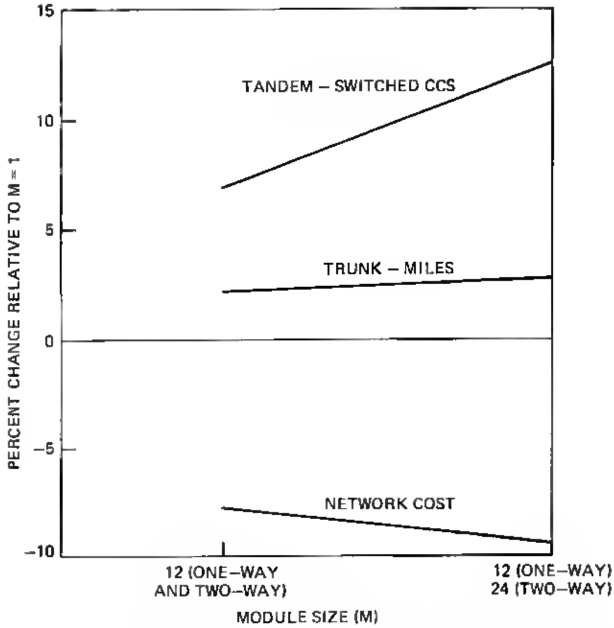


Fig. 7—Network cost, trunk miles, and tandem-switched load (one- and two-way groups).

thus exists to gain trunking efficiencies and to exploit potential noncoincidence by engineering ESS-to-ESS groups on a two-way basis.

The preceding results for one-way groups suggest 24 trunks as the proper module size for two-way groups, since the groups no longer come in pairs. Accordingly, the network was reengineered with two-way groups between pairs of ESS offices; these groups were assumed to be provided on dedicated digroups. All remaining groups were engineered on a one-way basis, as before. Two modular engineering alternatives were considered:

(*i*) All high-usage groups terminating on DCT or DIF in modules of 12 trunks.
(*ii*) One-way groups in modules of 12 and two-way groups in modules of 24.

Round-up thresholds were one-half module, as before.

Figure 7 shows the trunk miles, tandem-switched CCS, and network cost for these alternatives relative to cost-ratio engineering. We observe that the network cost is indeed lowest when two-way groups are sized in modules of 24 trunks, in spite of the increased trunk miles and tandem-switched load. These increases are more than offset by the

improved utilization of dedicated digroups and terminals resulting from matching the trunk group sizes to the fundamental multiplexing module.

4.3 Rounding thresholds

With module sizes of 12 and 24 trunks for one- and two-way groups, respectively, the round-up thresholds were varied to minimize the network cost. The optimal thresholds were found to be 7 trunks for modules of 12 and 13 trunks for modules of 24. The network cost, however, was reduced by only a fraction of a percent relative to thresholds of 6 and 12. Furthermore, sensitivity studies described in Section 5.1 revealed that the optimal thresholds can be larger or smaller than one-half module, depending on the particular network configuration, and that network cost is not very sensitive to these thresholds. Accordingly, round-up thresholds of one-half module are the best choice.

4.4 Administrative costs

Administrative costs of record-keeping, measuring, and servicing are incurred on a per-group basis, independent of trunk group size. (The three-trunk minimum for nonmodular high-usage groups is designed to prevent the establishment of small groups, whose administrative costs would exceed the realizable savings in network cost.) As the module size (and hence the round-up threshold) is increased, an increasing number of high-usage groups are disallowed, reducing the overall administrative costs of the network.

Figure 8 shows the reduction in the total number of groups in the network with one-way groups only. For modules of six trunks, there is no reduction, since the corresponding round-up threshold of 3 trunks is the same as the minimum group size. With a round-up threshold of 6 trunks (modules of 12), the total number of groups (and hence the administrative cost) is reduced by 26 percent. An additional reduction of 22 percent is achieved with a rounding threshold of 12 trunks (modules of 24). Comparison of the actual costs, however, reveals that this additional reduction in administrative costs is not sufficient to offset the increased network cost as shown in Fig. 5. We conclude that modular engineering results in administrative savings in addition to network savings, but that the administrative savings are not sufficient to affect the choice of module size.

4.5 Switch exhaust penalties

All network costs discussed to this point include the incremental cost of switching, in terms of the DIF termination cost, under the assumption that the No. 4 ESS has sufficient capacity to accommodate

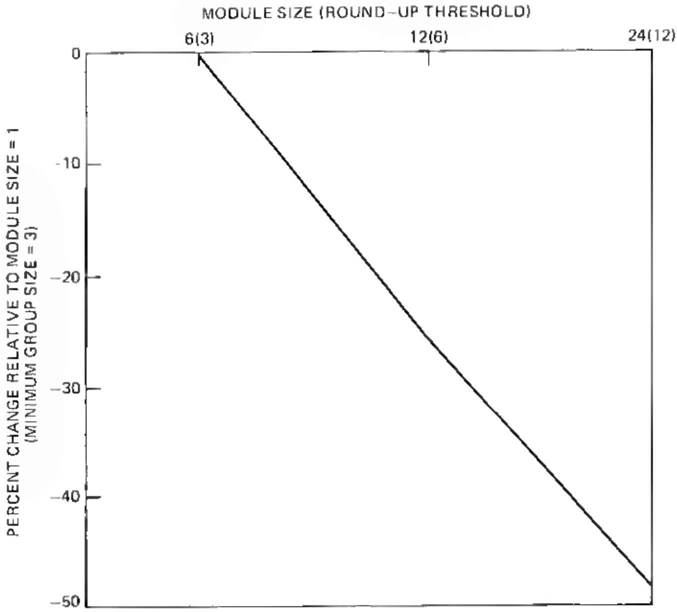


Fig. 8—Total number of groups.

the increased terminations resulting from modular engineering. (The No. 4 ESS is termination-limited.) We now examine whether the cost penalties for an accelerated switch exhaust are sufficient to offset the savings identified with modular engineering.

To determine the natural growth in tandem switching as a function of time, the network was engineered for loads projected 13 years into the future. Other network changes during that period of time, such as planned conversions of electromechanical offices to ESS, the introduction of a second No. 4 ESS tandem, area transfers, and rehomings, were also included.

Figure 9 shows the total number of T-carrier trunk terminations on the tandem switch(es) for the one-way group case with no modular engineering and with modules of 12. (The end-points are connected with straight lines for simplicity.) As observed earlier, modular engineering increases the number of terminations. The largest increase is about 5000 terminations, or slightly less than 5 percent of the capacity of a No. 4 ESS. For the given growth rate, this increase represents an advance of the switch exhaust date by at most 2 years (as measured by the horizontal distance between the two curves). Assuming discount and inflation rates of 12 and 7 percent, respectively, and a $4 million start-up cost of a new No. 4 ESS, the increase in present worth of first cost associated with this earlier exhaust is at most $352,000. Other penalties, such as trunk group splintering resulting from the addition

of a new switch, are typically smaller than the cost of a new switch alone[2] and have been neglected. The first cost saving due to modular engineering is (from Figure 6) \$3.9 million. Thus, the switch exhaust penalty has a small effect on the economics of modular engineering.

4.6 *Tandem switching constraints*

Where switching capacity is temporarily unavailable, or where it is desirable to reduce tandem loads to reduce the network's vulnerability to a tandem switch failure, the amount of tandem switching can be constrained by the use of an artificial switch cost in the cost-ratio computation.

To examine this approach, the switching cost used in the trunk engineering program was adjusted until the total tandem-switched load with modular engineering was the same as it was with no modular engineering and the original cost ratios. The actual network cost, as computed with the facility-cost program and with the original switching costs, was only about 0.3 percent higher than the cost of the unconstrained modular network. As the amount of switching—and hence its total cost—decreases, more traffic is carried on direct groups, resulting in an almost equal increase in the cost of facilities.

V. SENSITIVITY STUDIES

The robustness of the results to variations in several assumptions was tested in a series of sensitivity studies. We now describe some of these studies and their outcomes. The network as treated in the

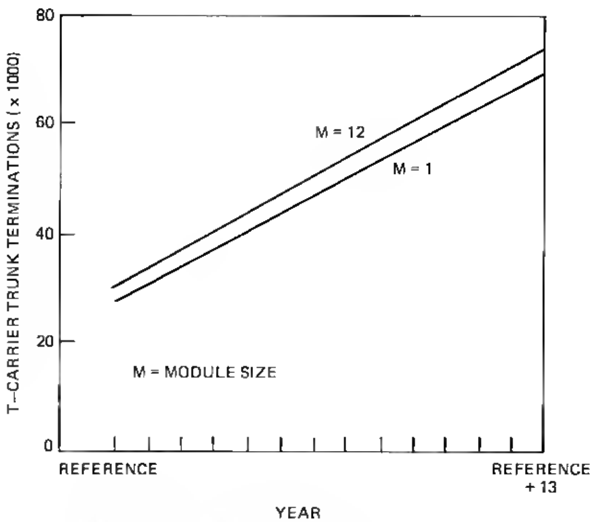


Fig. 9—T-carrier trunk terminations vs time.

preceding sections is termed the "reference" network. For the sake of brevity, all comparisons are made with the reference network with one-way groups only.

5.1 Network evolution

Several different evolutions of the Minneapolis/St. Paul network were examined. One, already described in Section 4.5, is a projection of the reference network 13 years from today. Others include models with separate local and toll trunks and switches, in which the local tandems are a No. 1A ESS and a crossbar tandem. The growth and possible evolutions of this particular network make it a rather generic model for all metropolitan networks.

In all cases, the optimal module sizes were the same, although the savings relative to the nonmodular alternatives varied between 4 and 9 percent. A typical result is shown in Fig. 10, for the future reference network.

5.2 Facility costs

Since a key parameter is the ratio P/q (per-system cost/per-circuit cost), P and q were varied about their nominal values. In the extreme, we increased q and decreased P by 50 percent each, well beyond the expected uncertainty in the nominal costs. (The nominal ratio P/q is thus decreased by two-thirds.) The result is also shown in Fig. 10. We note that, although the savings decrease, the optimal module size is still 12. Of course, if P is increased and q decreased, the savings increase.

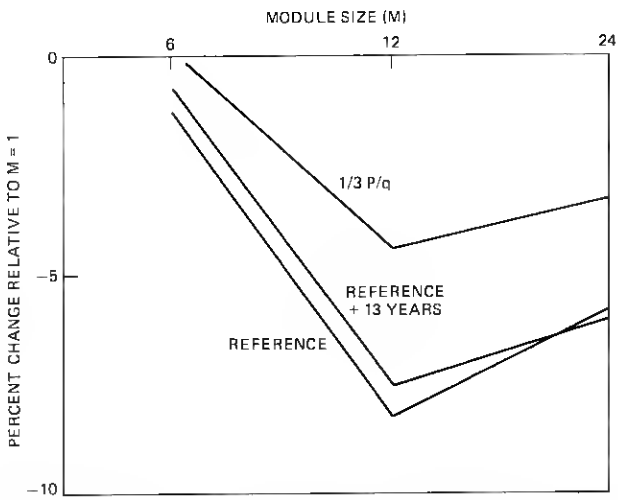


Fig. 10—Sensitivity of network cost savings to facility costs and to network evolution.

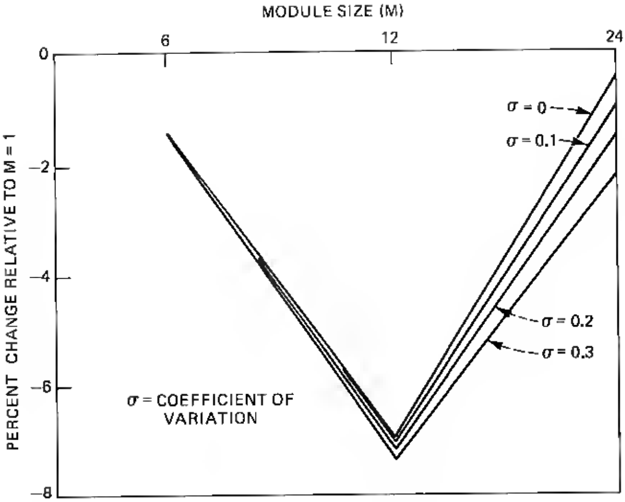


Fig. 11—Sensitivity of network cost savings to forecast error.

5.3 *Forecast error*

A network is typically designed to a forecasted set of loads. The actual loads to which this network is subjected after it is built may differ from the design loads as a result of errors in the forecast. The network must then be adjusted, or "serviced," to maintain a specified grade of service. To assess the impact of the added cost of servicing on the economics of modular engineering, a separate program was written to simulate the actual, realized loads as random perturbations of the design loads, to distribute these loads in the network as designed, and to resize all final trunk groups to the 1-percent blocking objective.*

Figure 11 shows the decrease in network cost relative to cost-ratio engineering as a function of module size, for values of the coefficient of variation of the forecast up to 0.3. Although the relative savings attributable to modular engineering tend to increase slightly with increasing forecast error, the shape of the curve remains substantially unaltered. Similar simulations revealed that forecast error also has a negligible effect on the choice of rounding thresholds.

VI. ACKNOWLEDGMENTS

The author wishes to thank Katherine Daigle for contributing a significant programming effort, Luis Suarez for carrying out the study

* As a result of some simplifying assumptions made in the load distribution program, the design network differs slightly from the reference network discussed earlier. These differences do not affect the conclusions.

of forecast-error sensitivity, and Fu-Tin Man and Antoinette Rule for providing the Minneapolis/St. Paul network model and data.

REFERENCES

1. S. W. Levine and M. A. Wernander, "Modular Engineering of Trunk Groups for Traffic Requirements," Fifth International Teletraffic Congress, New York, June 14–20, 1967.
2. R. B. Bertman, unpublished work.
3. C. J. Truitt, "Traffic Engineering Techniques for Determining Trunk Requirements in Alternate Routing Trunk Networks," B.S.T.J., *39*, No. 2 (March 1954), pp. 277–302.
4. M. Eisenberg, "Engineering Traffic Networks for More Than One Busy Hour," B.S.T.J., *56*, No. 1 (January 1977), pp. 1–20.
5. W. B. Elsner, "A Descent Algorithm for the Multihour Sizing of Traffic Networks," B.S.T.J., *56*, No. 8 (October 1977), pp. 1405–1430.